Astroparticle Physics

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Write your name, and S number on every sheet.
There are **4 questions** with a total number of marks: 15.
The use of graphical calculators is not permitted, only standard calculators are allowed.

Exercise 1 (3.5 marks)
Magnetic spectrometers are used to measure the properties of high energy particles.

a) (1 marks)
Derive the expression of the curvature radius of a particle of charge Ze, mass m and momentum p in a constant magnetic field of strength B, perpendicular to the particle's trajectory. Express the curvature radius in terms of the particle's rigidity.

b) (1.5 marks)
In the small angle approximation for θ, derive the formula that connects the sagitta s with the rigidity R, based on the sketch in figure 1.

c) (1 marks)
Discuss the purpose of magnetic spectrometers in the frame of astroparticle physics, and indicate the physical quantities that they can measure.

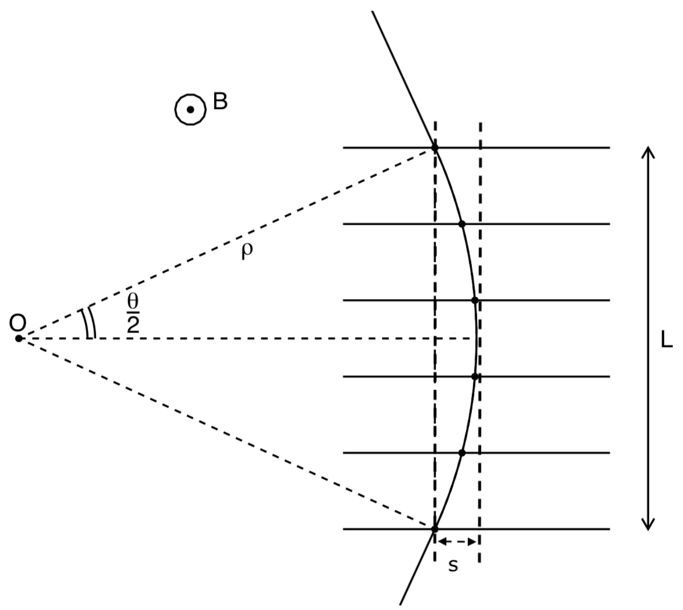

FIGURE 1. Sketch of the momentum measurement in a uniform magnetic field, perpendicular to the particle's momentum. L is the size of the detection region (permeated by the constant magnetic field of strength B), where the detection planes are also indicated by the horizontal lines. The track curvature radius is indicated by ρ, while θ is the deflection angle, and s is the track sagitta.

Exercise 2 (3 marks)
In the frame of the Heitler model for electromagnetic showers, the cascade development depends on the initial energy of the primary particle E_0, the radiation length X_0, as well as the critical energy E_c, which is (at least in first approximation) a property of the medium in which the shower develops.

a) (1 mark)
Describe the Heitler model for electromagnetic showers, naming the relevant processes and the limiting assumptions.

b) (1 mark)
Based on this model, derive the expression for the depth of the shower maximum X_{max}, as well as for the number of particles at the shower maximum N_{max}.

c) (1 mark)
Describe the basic processes for the ground-based detection of gamma rays: explain how the photons are detected, the concept of the light pool, and how these detectors work.

Exercise 3 (3.5 marks)

The first direct neutrino detection was done by observing the follow process:

$$\bar{\nu}_e + p \rightarrow n + e^+$$

Assuming that the neutrino is mass-less:

a) (0.5 marks)
Write down the expression of the total initial 4-momentum in the proton rest frame.

b) (0.5 marks)
Write down the expression of total 4-momentum after the interaction, in the Center of Mass frame.

c) (1.5 marks)
Derive the expression for the neutrino threshold energy for this process to occur, in terms of the masses of the other particles.

d) (1 mark)
Discuss how this process can be detected, and indicate which experimental device(s) can be used.

Exercise 4 (5 marks)
The IceCube experiment is a cubic kilometer-scale neutrino telescope, located deep under the ice of the South Pole. It consists of an array of 5160 light sensors, covering a volume of a cubic kilometer, from 1450 meters to 2450 meters depth. The sensors are distributed along vertical strings, deployed on a hexagonal grid with 125 meters spacing, each one housing 60 light sensors. The vertical separation of the light sensors is 17 meters. The density of ice is 0.92 g/cm^3, and refractive index of ice is $n = 1.309$.

Neutrinos are not observed directly, but via the detection of the Cherenkov light emitted in ice as charged leptons propagate faster than the light in the ice. However, the majority of the events recorded by IceCube are atmospheric muons, produced by cosmic ray interactions with the atmosphere.

a) (1 mark)
List and discuss the dominant processes that affect the propagation of muons in the Earth atmosphere.

b) (1 mark)
Neglecting the energy losses, show that $E_{min} = 2\ GeV$ is the minimum energy that a vertical muon, produced at height $h = 10.1\ km$, must have in order to reach the bottom of the IceCube detector. The muon lifetime is $\tau = 2.2\ \mu s$, and its mass is $m_\mu = 105\ MeV/c^2$.

c) (1 mark)
Show that the minimum energy that a muon must have in order to induce the emission of the Cherenkov radiation in ice is: $E_{thr} = 163\ MeV$.

d) (1 mark)
Consider a vertical muon of energy $E = 2\ GeV$ entering the detector from the top. Calculate the length of the path in the ice along which Cherenkov radiation is emitted. Consider a constant energy loss via ionisation $\frac{dE}{dx} = 2\ MeV\ g^{-1}\ cm^2$, and neglect the energy losses via Cherenkov emission.

e) (1 mark)
Assume that the cross section per nucleon for muon-ice interaction is $\sigma_{\mu-ice} = 400\ mb$ ($1\ b = 10^{-24}\ \mathrm{cm}^2$). Calculate the mean free path for a muon moving through the IceCube detector.
Use $N_A = 6 \cdot 10^{23}\ \mathrm{mol}^{-1}$.

Solutions

Exercise 1

a) (1 marks)
The curvature radius ρ for a particle with charge Ze and momentum p in a magnetic field B is given by:

$$\rho = \frac{p}{ZeB} = \frac{R}{cB}$$

And it is obtained by equating the Lorentz force to the centripetal force.

b) (1.5 marks)

it can be derived by considering that:

$$\frac{L}{2} = \rho\ sin\theta/2 \simeq \rho\ \theta/2 \to \theta = \frac{L}{\rho}$$

The sagitta is then derived considering that:

$$\rho = \rho cos\frac{\theta}{2} + s \to s = \rho(1 - cos\frac{\theta}{2}) \simeq \rho\frac{\theta^2}{8} = \rho\frac{L^2}{8\ \rho^2} = \frac{L^2}{8\ \rho}$$

Considering that $\rho = \frac{R}{cB}$, we can rewrite the previous formulas as:

$$s = \frac{cBL^2}{8R} \to R = \frac{cBL^2}{2s}$$

c) (1 marks)
Magnetic spectrometers are position-sensitive devices used in astroparticle physics for cosmic ray measurements in the GeV to TeV range. The charged particle rigidity and charge sign are measured by sampling the particle track bending in a constant magnetic field of known intensity, via the measurement of the sagitta, as discussed above.

The cosmic ray charge magnitude (the atomic number Z) can also be measured by detecting the energy loss per traversed unit length, that is proportional to Z^2. The measurement of the particle momentum, combined with the particle velocity (measured with other detectors, like Cherenkov detectors or Time of Flight systems) provides the measurement of the cosmic ray mass $m = \frac{RZ}{\gamma\beta}$.

Exercise 2

a) (1 mark)
Relevant processes:

- bremsstrahlung for electrons (and positrons)
- pair production for photons.

Limiting assumptions:

- The toy model does not take into account that the shower is intrinsically a random process, in which electrons can eventually radiate several photons per interaction.
- The cascade development suddenly stops after the critical energy is reached, while in reality the process is clearly smoother.
- Both photons and electrons (or positrons) interact exactly after each radiation length X_0, while the X_0 for electrons and photons has different meaning.

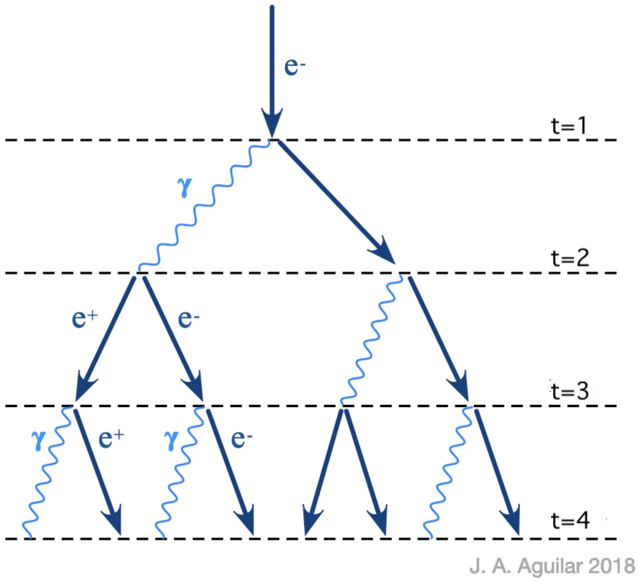

FIGURE 2. Toy model for electromagnetic showers, courtesy of J. A. Aguilar.

b) (1 mark)
Based on this model, derive the expression for the depth of the shower maximum X_{max}, as well as for the number of particles at the shower maximum N_{max}.

It is useful to introduce the dimensionless variable $t = \frac{x}{X_0}$, indicating the number of radiation lengths. Figure 2 shows a schematic representation of an electromagnetic shower initiated by an electron. The electron and positron will each have energy $\frac{E_0}{2^2} = \frac{E_0}{4}$. After t radiation lengths we shall have 2^t secondary particles, each one with energy $\frac{E_0}{2^t}$.

The shower development stops when the particles reach the critical energy:

$$E(t) = \frac{E_0}{2^t} = E_c \tag{1}$$

At this stage ionization loss become dominant and no further pair production or bremsstrahlung process is allowed. In air, $E_c = 85\ MeV$. This point is the shower maximum, reached after the following number of radiation lengths:

$$t_{max} = log(\frac{E_0}{E_c})/log2 \tag{2}$$

corresponding to the following number of particles:

$$N_{max} = \frac{E_0}{E_c} \tag{3}$$

Such that the depth at which the shower reaches its maximum, measured in $g\ cm^{-2}$, is given by:

$$X_{max} = X_0\ log(\frac{E_0}{E_c})/log2 \tag{4}$$

c) (1 mark)
The Earth atmosphere is opaque to γ rays, and for this reason they cannot be

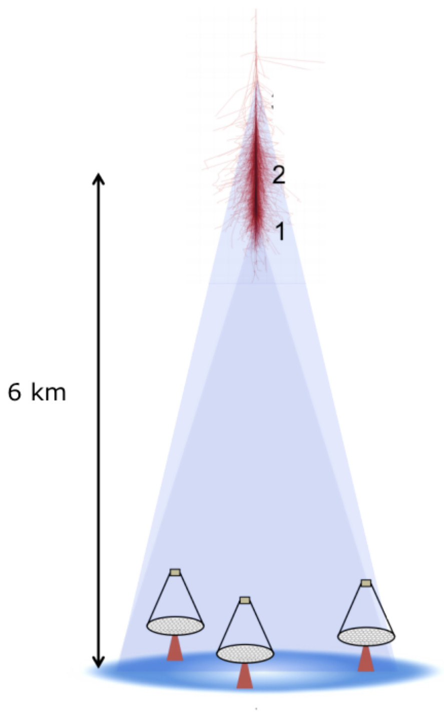

FIGURE 3. An illustration of the stereoscopic imaging technique. A gamma ray generates an electromagnetic cascade in the Earths atmosphere, which generates Cherenkov radiation illuminating an area, called the "light pool", on the ground. Telescopes within this light pool are used to form an image of the shower, which allows reconstruction of the arrival direction of the incident primary photon. You can neglect the numbers "1,2" indicated in the shower.

observed directly from the ground. When γ rays arrive at the top of the atmosphere, they generate a cascade of particles, called electromagnetic showers, whose development is fostered by two main physical processes: the pair-production and the bremsstrahlung from the electrons and positrons in the presence of (atmospheric) nuclei. The particles produced in the cascade, called the secondary particles, also travel through the atmosphere: if they travel faster than the speed of light in the medium, they induce the emission of Cherenkov radiation. The Imaging Atmospheric Cherenkov Telescopes (IACTs) are ground-based detectors, made of large mirrors and high-speed cameras (equipped with fast light detectors called photomultipliers tubes) used to detect the flash of light, and image the cascade generated by the γ rays. This technique is used to study γ rays with energies above a few tens of GeV, and it is efficient up to hundreds of TeV.

A sketch of the detection principle is shown in figure 3. These detectors are operated only in clean, moonless nights and have precision in the position better than one degree, so they can be pointed to the sources to be used to perform γ-ray astronomy.

Exercise 3

The first direct neutrino detection was done by observing the follow process:

$$\bar{\nu}_e + p \rightarrow n + e^+$$

Assuming that the neutrino is mass-less:

a) (0.5 marks)
Assuming that the neutrino is mass-less (and using c=1) the neutrino energy is given by:

$$E_\nu = \sqrt{m_\nu^2 + p^2} = p$$

For the proton, in the proton rest frame, we have:

$$E_p = \sqrt{m_p^2 + p^2} = m_p$$

The four-momentum for the two particles is given by:

$$q_\nu = (E_\nu, \vec{p}) = (p, \vec{p})$$

$$q_p = (E_p, \vec{p_p}) = (m_p, \vec{0})$$

Thus total initial 4-momentum is given by:

$$q_{TOT} = (m_p + p, \vec{p})$$

b) (0.5 marks)
Note that in the center of mass (COM) frame there is no net momentum. So we can simply add the two 4-momenta for the neutron and the positron. Note that in their energies we do have the momentum term in there, COM-frame does not mean that both particles are at rest.

$$q_{COM} = (E_n + E_{e^+}, 0)$$

where,

$$E_i = \sqrt{m_i^2 + p_i^2}$$

c) (1.5 marks)
The threshold condition is:

$$(q_p + q_\nu)^2 \geq (m_n + m_e)^2 \rightarrow m_p^2 + 2E_\nu m_p \geq (m_n + m_e)^2$$

Thus the neutrino energy is given by:

$$E_\nu = \frac{(m_n + m_e)^2 - m_p^2}{2\ m_p}$$

d) (1 mark)
This process can be detected if neutrinos are interacting with the protons of a dielectric medium, and the corresponding electrons have enough energy to induce the emission of the Cherenkov radiation. The Cherenkov radiation can be detected using light sensors, like the PMTs, as it is done by SuperKamiokande or IceCube.

Exercise 4

a) (1 mark) Dominant processes limiting the muon's travel time:
   - Decay of the muon.
   - Muon-air molecule interactions.
   - Brehmsstralung.

b) (1 mark) The muon has $\gamma = \frac{E_{min}}{m_\mu} = \frac{2000}{105} \approx 19.05$. So it travels a distance of

$$\gamma \tau c = \frac{E_{min}}{m_\mu} \tau c \approx 12.5\ km \approx h + d = (10.1 + 2.45)\ km.$$

Where $d = 2.45\ km$ is the depth of the bottom of the detector.

c) (1 mark) In order to generate Cherenkov light, the speed of the particle must exceed the speed of light in the medium: $v > c_n = c/n \rightarrow \beta > 1/n = 0.764$. Since $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ by definition, this gives a threshold value for γ of 1.55. The correspondent energy threshold is $E_{thr} = \gamma m c^2 = 162.7$ MeV ≈ 163 Mev.

d) (1 mark) The Cherenkov emission stops when $E_\mu = E_{thr}$. So it can lose $\Delta E = E_{min} - E_{thr} = 2000\ MeV - 162.7\ MeV = 1837.3\ MeV$. This happens in a distance of $\Delta E / \frac{dE}{dX} = 1837.3/2\ gcm^{-2} = 918.65\ gcm^{-2}$. This is a length of $918.65\ gcm^{-2}/0.92\ gcm^{-3} = 998.5\ cm \approx 10\ m$.

e) The mean free path is $\lambda = \frac{1}{\rho_n \sigma} = \frac{m_M}{N_A \rho \sigma} = \frac{18}{6.02 \cdot 10^{23}\ 0.92\ 0.4 \cdot 10^{-24}}$ cm ≈ 81 cm, where ρ_n is the nucleon number density and $m_M = 18$ g/cm^3 is the molar mass of water.